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Shapes of heaps and in silos

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Abstract. Experimental investigations on the shape of a heap formed in a Hele Shaw cell either on a flat base or in a two-dimensional silo are presented. We have focused our attention on the shape dependence on mass flux and initial energy of particles poured into the cell. Two kinds of granular media are considered: glass beads and sand and we shall point out their different behaviors. We described the variations of the angle of repose and of the size of the tail as a function of the experimental parameters. We also report the time evolution of the angle of repose during the formation of the heap.

PACS. 61.43.Gt Powders, porous materials – 81.05.Rm Porous materials; granular materials – 83.70.Fn Granular solids

1 Introduction

The shape of granular heaps strongly depends on the formation processes. Generally a heap surface consists of three different areas: a rounded part at the top due to the impacts of the falling particles, a central linear region where the angle of repose is commonly defined and an other rounded part at the bottom (with an opposite curvature as compared to the top) known as the tail of the heap. From these three parts, the most studied is the linear region, *i.e.* the angle of repose, because of the hope to directly relate it to the microscopic properties of particles (size, friction and restitution coefficients). One knows that also the formation process of the heap influences the angle of repose [1]. We will consider here the initial energy of particles and the mass flux but it is important to note that other effects, like humidity, also affect the shape.

A previous study on the shape of a glass beads heap formed in a two-dimensional silo has shown important changes in the shape as a function of the energy of particles poured on top of the heap [2]. We have investigated the dependence of the size of the tail as a function of the energy of particles for a fixed particles flux. Using a kink model based on the vertical translational invariance of the surface observed experimentally, we have obtained an equation for the surface where the tail was described by $\gamma \ln(L/x)$, where L is the size of the silo. This quantity can also be retrieved from the continuum theory of Boutreux and de Gennes [3]. We found that the parameter γ, representing the size of the tail (*i.e.* the deviation from the straight line), was proportional to the energy of particles having, for zero energy a value between 2 and 3 particles diameters, in agreement with results obtained by Alonso and Herrmann [4] for heaps on a table. But no investigations were made neither on the dependence of the flux of particles nor on another type of granular medium, for example sand, which has a different behavior from glass beads [1].

Recent theoretical and numerical works have investigated the shape dependence of a heap. Karolyi *et al.* [5] have used a cellular automaton to calculate the surface profiles while filling a silo with a weak particle flux as a function of the restitution coefficient of particles. Boutreux *et al.* [6] have, on the other hand, made a theoretical study on the influence of the particle flux on the angle of repose of a heap constructed in a 2D silo. The authors found that the angle of repose, θ, of a heap formed in a 2D silo of size L, using a particle flux, W, behaves as $\theta(W = 0) - W/L$: the variations of angles being of the order of the degree.

In this paper, we present an experimental study on the dependence of the shape of a granular heap on the initial energy and flux of particles. We will consider two cases in a vertical Hele Shaw geometry: a heap on a flat base and a two dimensional silo and we will use two kinds of granular media: glass beads and sand.

2 Experimental setup

Experiments are performed within an vertical Hele Shaw cell (20×30 cm) having a fixed thickness of 5 mm (*cf.* Fig. 1), the granular medium being injected close to the left wall. For this thickness, the rigid walls do not play a significant role on the angle of repose [4]. The cell thickness is larger than about 20 grain diameters, so that the local grain mobility is essentially three dimensional. The setup is however two dimensional since there is no degree of freedom in the shape of the surface in the direction perpendicular to the wall. The case of a 2D silo is realized

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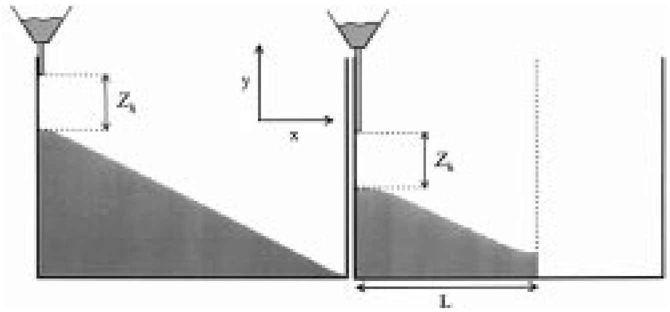

Fig. 1. Experimental Hele Shaw cell (thickness 5 mm). A rigid rod (dashed line) can be inserted into the cell to consider the 2D silo geometry. Particles flow from the funnel through an outlet of variable diameter allowing to change the flux, W, of particles. The energy of the particle is given by the falling height, Z_{h} (measured in cm), above the heap.

by inserting a rod into the cell at a variable distance L from the left wall. This procedure allows us to also change the size L of the silo. In these two geometries, heaps are constructed using glass beads or sand, both with an average particle diameter of $a = 250$ μm. The two experimental parameters, initial particle energy, Z_{h}, and flux, W, are monitored as following. Particles in the funnel are poured into the cell through an outlet of variable diameter which controls W allowing for the values 0.32 g/s, 0.6 g/s, 1 g/s, 2.2 g/s and 5.25 g/s. The pouring energy is controlled by the falling height, Z_{h}, of the particles above the heap and is therefore measured in cm. This height is kept constant while filling the cell by moving the outlet up with the same velocity as the growth of the top of the heap. In this paper, we will measure the energy through Z_{h} which ranges from 0 (particles carefully deposited on the top) to approximately 10 cm. The experimental surfaces of heaps are retrieved by image analysis. We have two possible ways to proceed. In some cases, we make snapshots of the heaps in the static regime (*i.e.* without flow of particles). Several pictures are then averaged to obtain the shape (angle of repose, tail) as a function of W and Z_{h}. The other possibility is to make a video recording of the heap's growth to study the modifications of the angle of repose as a function of time.

3 Two-dimensional silo

In this part, we will focus on the 2D silo geometry and first give a qualitative description of the role of initial energy and flux of particles on the shape of a heap constructed with glass beads. In Figure 2 are shown experimental pictures of the surfaces of heaps for a size of $L = 9$ cm. In the left pictures, the flux is kept constant ($W = 1$ g/s) and the initial energy of particles is changed while on the right side, the energy is fixed to $Z_{\mathrm{h}} = 4$ cm and the flux is varied. The right lower corner corresponds, in all images to the same point. The increase of Z_{h} or W gives a flatter surface and an increase of the size of the tail. We have also observed experimentally that the shapes of the

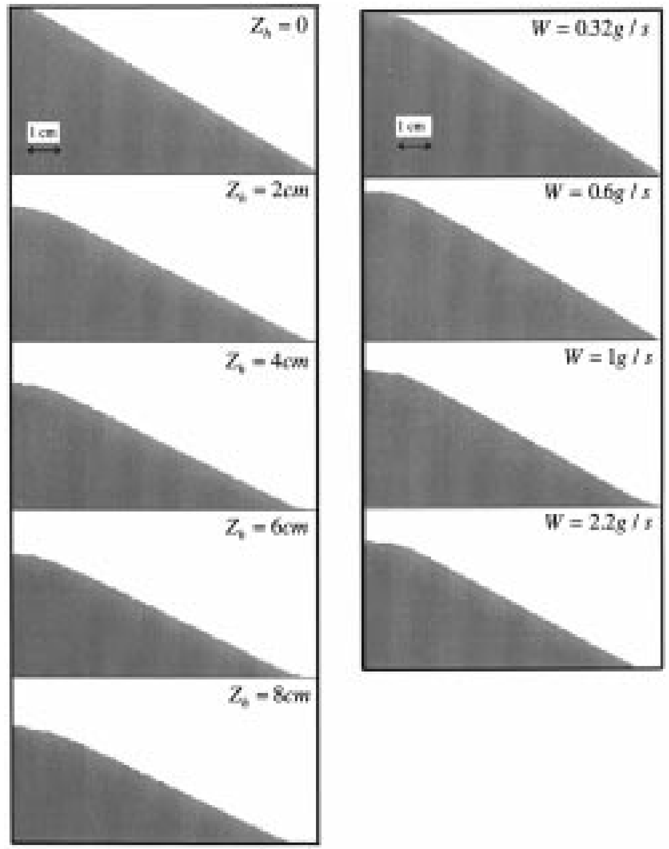

Fig. 2. Experimental pictures of the surfaces of heaps constructed with glass beads (250 μm) in a 2D silo geometry of size (distance between left and right wall) $L = 9$ cm. Left picture: effect of initial energy measured by the falling height Z_{h} of the particles at fixed flux $W = 1$ g/s. Right picture: effect of flux W at fixed $Z_{\mathrm{h}} = 4$ cm. Each picture is of the same size and the lower right corner corresponds always to the same point.

rounded part on top of the heap are nearly independent on Z_{h} and W. For different Z_{h} and W, the upper parts of heaps can be superposed on a length of approximately 2 cm just by shifting vertically the surfaces. On the contrary, the linear regions of the heaps present an angle θ which depends both on Z_{h} and W and, in fact, also on L. In a 2D silo, the angle of the linear part and the tail are characteristic of the shape and we will describe in the following the role played by the energy and the flux of particles.

In certain experimental conditions, we have observed an other interesting phenomenon, presented in Figure 3: close to the tail, one can note a local increase of the slope. This effect is perfectly reproducible for given experimental conditions and appears for large L and small Z_{h}, after some minimum height is reached. The phase diagram of this effect is also shown in Figure 3. In area I, the increase of the slope is not observed while it exists in area II. We will discuss later that point but a possible way to explain this effect is to relate the increase of the slope to an increase of the bulk density [7]. To determine experimentally the density of the a heap, we have filled the silo

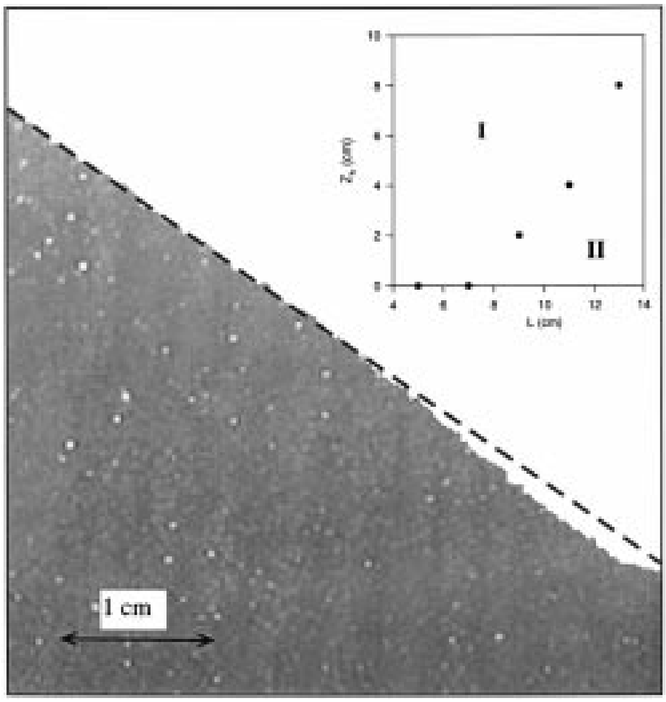

Fig. 3. Zoom of the bottom part of the shape of a heap of glass beads obtained in experimental conditions located in the region II of the phase diagram (inset). In area I, the increase in slope close to the tail is not observed. This effects may be related to the increase in density of particles.

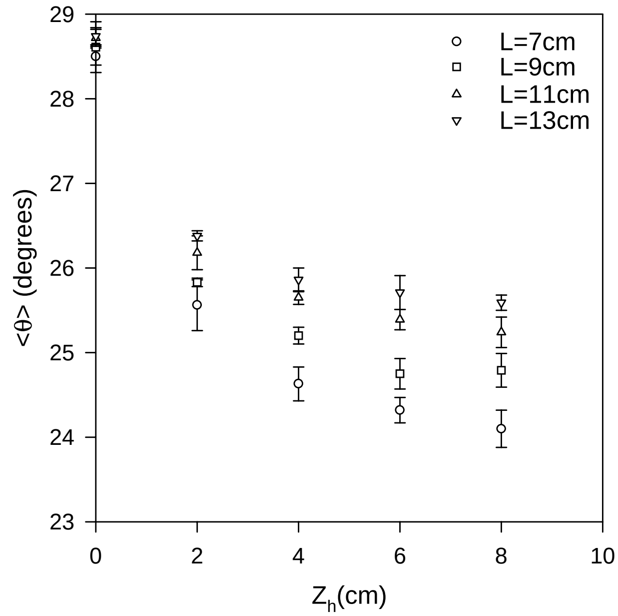

Fig. 4. Experimental values for the average angle, θ, of the linear region of the heap, as a function of energy Z_{h} for different sizes of silo L. $Z_{\mathrm{h}} = 0$ means that particles are carefully deposited on the top of the heap and it is the only case where θ is independent of L within the errors bars.

with a known mass of glass beads having a mass density of about 1.8 g/cm^3. The filled volume (*i.e.* the volume of the heap) is measured directly on pictures retrieved from experiments. We have determined densities, for $Z_{\mathrm{h}} = 0$ equal to 1.62 ± 0.03 g/cm^3 (mass density times volume fraction) while for non zero energy, the density was found to be almost the same and equal to 1.83 ± 0.01 g/cm^3. These measurements are not very precise but the difference is significant; it shows that the external perturbations (impact of the particles on the surface for $Z_{\mathrm{h}} \neq 0$) are able to change the inner structure of the heap. An other possible explanation is to consider a plastic deformation of the bulk due to its own weight.

3.1 Effect of the energy of particles

For a fixed flux, $W = 1$ g/s, we have constructed heaps with several sizes of the silo, L, and different initial energy of particles, Z_{h}, using glass beads. The average angle, θ, of the linear part of the heaps (retrieved from a linear regression of the experimental surface) as a function of Z_{h} is presented in Figure 4. For a fixed L, we find again what was earlier observed in Figure 2: θ decreases with increasing energy. Experimentally, we have the impression that θ saturates for high initial energy to a value which is L-dependent. This effect is due to the size limitation of the heaps constructed. In fact, if we assume a very large heap, the angle of repose should not depend on the initial energy of the particles: the particles will bounce on the surface of the heap until they loose their initial energy and then they reach a steady state energy. The length of the surface has to be large enough, in order that particles will attain their steady state. That means that after a characteristic length, the angle of repose will not depend any more on the initial energy of the particles. This length should depend on the ratio L/a for a given initial energy and material. This situation is not achieved in our experiments and only for $Z_{\mathrm{h}} = 0$, the formation process of the heap is identical for each L.

The dependence of the size of the tail as a function of the energy of particles has been presented in a previous paper and we just give here the relevant results. With the vertical translational invariance of the surface observed experimentally for fixed L and Z_{h} we have developed a kink model for the heap growth to derive the slope of the surface close to the tail. The difference between the real shape and the straight line, given by the angle of repose, is a logarithmic term: $\gamma \ln(L/x)$. γ represents the size of the tail and its dependence *versus* Z_{h} is given in Figure 5. The results are obtained by a fit of the experimental surfaces (see inset of Fig. 5). The behavior of γ is approximately linear with an intercept at $Z_{\mathrm{h}} = 0$ between 2 and 3 times the particle diameter, a. The divergence observed for $x \rightarrow 0$ has a cut-off when it becomes one particle diameter wide [2]. The points at $Z_{\mathrm{h}} = 0$ only represent the fact that we did not observe experimentally a sufficiently large tail to make a reasonable fit. This description of the tail works well if Z_{h} is not too high, typically for Z_{h} lower than 10 cm and sizes of silo large enough (above $L = 7$ cm).

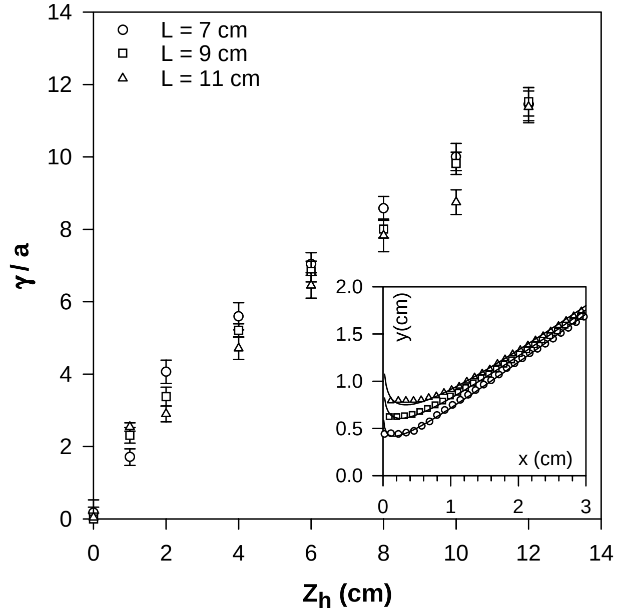

Fig. 5. Parameter γ describing the size of the tail (for glass beads of size $a = 250$ μm) as a function of the energy of the particles $Z_{\rm h}$, for different sizes of the 2D silo L. The behavior is linear with an intercept at $Z_{\rm h} = 0$ ranging from 2 to 3 diameters of particles. Also plotted in the inset is the logarithmic fit (solid lines) of the tail for $L = 7$ cm and (from bottom to top) for $Z_{\rm h} = 2$, 4 and 6 cm. The divergence at $x = 0$ is of the order of some particle diameters.

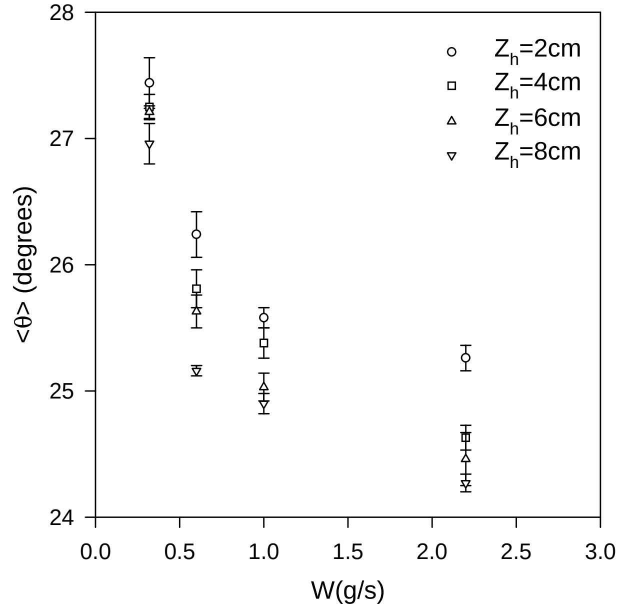

Fig. 6. Average angle of the linear region of a heap constructed in a 2D silo with glass beads ($a = 250$ μm) as a function of the particle flux W for different energies $Z_{\rm h}$ (size of the silo $L = 11$ cm). The angle presents a sharp decrease with W and tends to saturates for higher W.

3.2 Effect of the flux of particles

The average angle, θ, defined in the linear region of the heaps as a function of W is plotted in Figure 6 for different $Z_{\rm h}$ and $L = 11$ cm. θ is a decreasing function of W, the more pronounced the larger the energy. The theoretical linear behavior of Boutreux *et al.* seems to be valid only for weak fluxes. It seems that θ could saturate for higher W but we are not able to show this because, for our highest W, the flux is so strong that the heap exhibits different shapes from the typical ones presented in this paper (*cf.* Fig. 7). In such cases, no linear region can be clearly defined and the local angle of the surface decreases as one goes from top to bottom(*cf.* Fig. 3).

The size of the tail as a function of W is plotted in Figure 8 for different $Z_{\rm h}$ and two sizes $L = 7$ cm and $L = 9$ cm. We clearly see that γ saturates with W and faster the smaller the size of the silo. This means that, for a given $Z_{\rm h}$, above a critical flux, increasing W does not increase the size of the tail. A possible explanation is that for small L and/or large $Z_{\rm h}$, the glass beads rolling on top of the heap reach the bottom of the pile with a large kinetic energy. In consequence, a fraction of the particles (which can be large for very small L and very large $Z_{\rm h}$) are reflected by the right wall of the silo and then accumulate at the bottom of the heap. This is also probably why the logarithmic shape of the tail (retrieved from a kink model) is no longer valid for such experimental conditions.

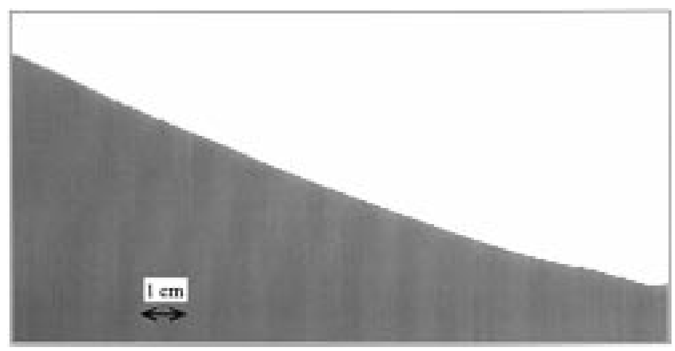

Fig. 7. Shape of a glass beads heap in a 2D silo ($L = 15$ cm) and an initial energy of particles $Z_{\rm h} = 2$ cm. Effect of a strong flux $W = 10$ g/s. The slope of the heap is continuously changing from top to bottom.

3.3 Comparison with sand

It is well-known that a granular medium consisting of sand has a different behavior than one made of glass beads. So we have repeated the same experiments presented above with a sand having an average grain size $a = 250$ μm. For such medium, the increase of the slope close to the tail observed under certain circumstances for glass beads is weaker. We could, as for glass beads, invoke a weak densification of a heap made of sand but a better explanation may be the relaxation time for the internal structure. The rolling grains on top of the heap act like a shear layer on the surface which gives possible orientation to the structure. The time needed for the structure to relax and densify might be larger for sand than for glass beads and it is

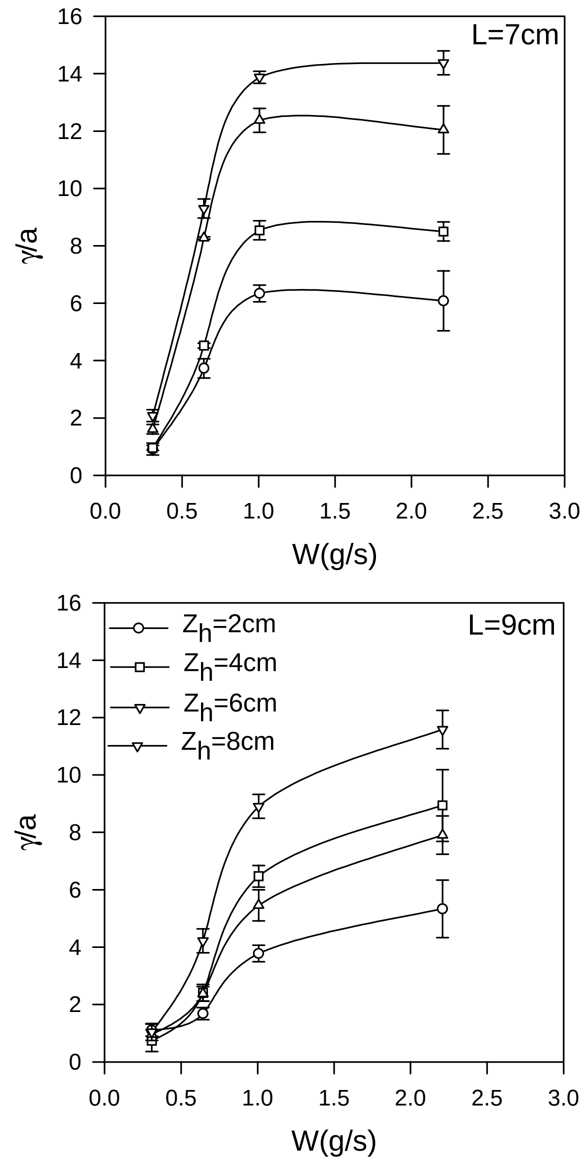

Fig. 8. Parameter γ, expressed in particles diameters, a, describing the size of the tail *versus* the flux of particles W for different energies Z_{h}. Two sizes of silos are considered $L = 7$ cm and $L = 9$ cm. γ saturates with W the faster the smaller L. The solid lines connecting the experimental points are given to improve the visualisation of the saturation of the size of the tail at high flux W.

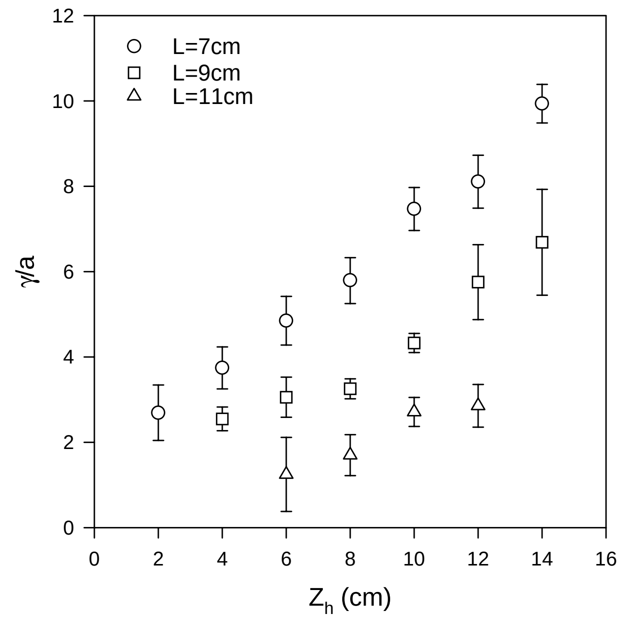

Fig. 9. Same curve as Figure 5 but for a medium made of sand (size $a = 250$ μm).

possible that, in our experiments the formation times of heaps are too short compared to this compaction time.

Energy and flux have the same effect on sand heaps as on heaps of glass beads. We will not present the curves of θ as a function of Z_{h} and W but the general appearances are the same as the ones of Figures 3 and 6. Nevertheless, the relative decrease in the value of the angle is lower, for example only one degree difference between $W = 0.32$ g/s and $W = 2.2$ g/s and about 2-3 degrees between $Z_{\text{h}} = 2$ cm and $Z_{\text{h}} = 8$ cm. The small decreases observed for sand are not surprising because of the lower restitution coefficient of grains in comparison with glass beads: the particles loose their initial energy mostly on the top of the heap. So, for sand, the flow of particles on the surface is weaker and the erosion phenomenon which tends to give flatter surfaces is less pronounced. The determination of the parameter γ (the size of the tail) for sand heaps in a 2D silo as a function of Z_{h} is presented in Figure 9. Missing experimental points are due to the size limitation of our experimental cell. The behavior of γ is, here, dependent of the size of the silo L as opposed to the case of glass beads (*cf.* Fig. 5). The low values obtained for $L = 11$ cm are due to the very small tails which are difficult to observe directly in experiments. A proportionality between γ and Z_{h} is not evident and we have also actually no explanation for the dependence on the silo size.

4 Time dependence of the angle of repose

We will now investigate the time dependence of the angle of repose of a heap constructed in a Hele Shaw cell on a flat base. We built heaps with glass beads and sand and took snapshots of the pile at fixed time intervals, the flow of particles being kept constant. In Figure 10 are plotted the successive shapes of a heap made with glass beads, retrieved from experimental pictures, for a flux $W = 2.2$ g/s and an initial energy $Z_{\text{h}} = 6$ cm. Each curve is separated by a time interval of 6 s. We note the modifications of the shape of the tail, as the pile grows, and also the appearance of the local increase in slope at the bottom like in the 2D silo case. The situation is different for sand. Except at the very beginning, the heap exhibit always the same shape and the tail is of the same size. The increase of the slope at the bottom is not obvious. The successive different shapes exhibited by the heap of glass beads can be qualitatively explained and related to the existence of different construction processes. If the size of the heap

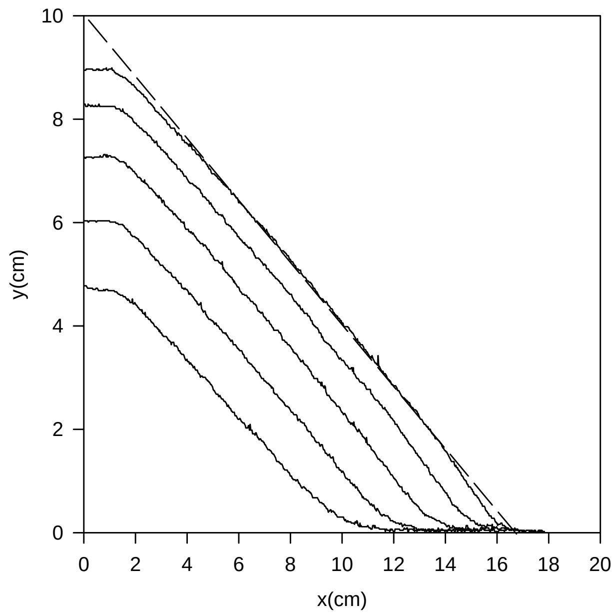

Fig. 10. Shape of a growing heap of glass beads as a function of time for an energy of $Z_{\rm h} = 2$ cm and a flux $W = 2.2$ g/s. Each curve is separated by a time interval of 6 s. Note the change in the shape and size of the tail and the appearance of a larger slope at the bottom of the pile once the pile exceeds a given size. The dashed line is a guide to the eye.

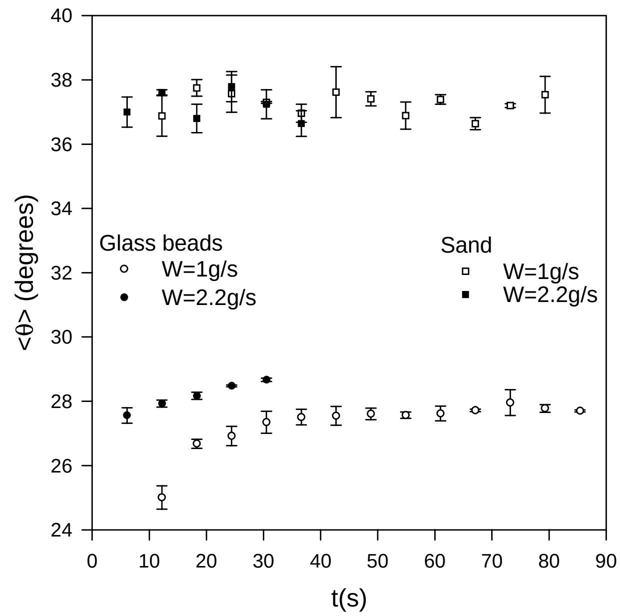

Fig. 11. Angle of repose as a function of time from a growing heap ($Z_{\rm h} = 2$ cm) retrieved from experimental surfaces as the ones presented in Figure 10. For sand, the angle is independent on time (or size of the heap) and on the flux W. For glass beads, the increase of the angle could be related to an increase of the bulk density inside the heap.

is small enough then, for given experimental conditions (W and $Z_{\rm h}$), the particles can easily reach the bottom of the heap because of their initial energy and to the small distance, between top and bottom. In that case, they accumulate at the bottom and give the large tail observed. On the contrary, if the size of the heap is large enough, the particles loose almost all their initial energy (because of inelastic collisions on the surface) before reaching the bottom and then just go down hill by rolling over the surface. There might exist a critical size of the heap, depending on experimental parameters, where this change of behavior occurs. In the case of sand, the grains loose more rapidly their initial energy and this explains that the shape of the heap is always identical. The increase of slope is not visible probably because of the same reasons invoked for the 2D silo.

We have determined the angle of the linear region as a function of time. Results are plotted, both for sand and glass beads, in Figure 11 for two fluxes $W = 1$ g/s and $W = 2.2$ g/s and for $Z_{\rm h} = 2$ cm. For sand, the angle is independent of time and of W. The apparent oscillations of the values after a time of 40 s are only due to the automatic treatment of the pictures. It depends whether an up going kink is present on the surface or not. If yes, the automatic evaluation of the angle gives a smaller value. On the contrary, the angle of a heap made of glass beads, rapidly increases at the beginning and after a given time (or a given size of the heap) saturates to a value approximately of 27.2 degrees almost close to the value determined for zero energy. The points obtained, for glass beads, at $W = 2.2$ g/s can be superposed to the values at $W = 1$ g/s if the angle is plotted as a function of the height of the pile. The increase of the average angle of the heap constructed with glass beads could again be related to a densification of the bulk. Future works will be done in tracking marker particles through the heap in order to quantify the densification process.

5 Conclusion

We have experimentally studied the dependence of the shape of a heap, constructed with sand or glass beads on a flat base and in a 2D silo, on the initial energy $Z_{\rm h}$ and flux W of particles. The angle θ defined on the linear region of the heaps was found to decrease while the size of the tail increases when increasing the energy and the flux of particles. If these experimental parameters are not too large, the tail can be described by a logarithmic term $\gamma \ln(L/x)$ where γ represents the size of the tail. The main results obtained are the following:

Heaps in 2D silo

• For glass beads, θ is found to be a decreasing function of $Z_{\rm h}$ and the more the smaller size of the silo L. This effect is due to the relative small sizes of heaps constructed in our experimental cell. θ is also a decreasing function of W: for weak fluxes, θ is linear in W. γ is proportional to $Z_{\rm h}$ and independent on L. On the contrary, γ saturates for large W, the more rapidly the smaller L.

• For sand, the behaviors are identical except that the decrease of θ is weaker. γ *versus* Z_h has a more complicated behavior and is L-dependent.
• Under certain experimental conditions (small Z_h and W), we have observed an increase of the local slope of the surface close to the tail. This increase being less pronounced for sand heaps and could be related to an increase of the bulk density. The different behavior between sand and glass beads may be attributed to the different relaxation time for the structure to densify because of the shear due to the moving particles on the heap surface.

Time dependence of the angle of repose

• For glass beads heaps, θ is found to be dependent on the size of the heap: it increases continuously and saturates. An increase of the average density of the heap can be invoked to explain these results. For sand heaps, this effect is not observed.

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